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22MIA/MAR11

## First Semester M.Tech. Degree Examination, Jan./Feb. 2023 Applied Mathematics

Time: 3 hrs.

Max. Marks: 100

*Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.*

*2. M : Marks, L: Bloom's level, C: Course outcomes.*

Module - 1			M	L	C
Q.1	a.	Solve the system of equations $x + 2y + 3z = 5$ , $2x + 8y + 22z = 6$ , $3x + 22y + 82z = -10$ using Cholesky method.	10	L3	CO1
	b.	Find the inverse of the matrix using partition method $\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}$	10	L3	CO1
<b>OR</b>					
Q.2	a.	Solve the system of equations: $x_1 + x_2 + x_3 = 1$ , $4x_1 + 3x_2 - x_3 = 6$ , $3x_1 + 5x_2 + 3x_3 = 4$ by triangularization method.	10	L3	CO1
	b.	Apply Gauss-Seidel iterative method to solve the system of equations. $10x_1 - 2x_2 - x_3 - x_4 = 3$ , $-2x_1 + 10x_2 - x_3 - x_4 = 15$ $-x_1 - x_2 + 10x_3 - 2x_4 = 27$ , $-x_1 - x_2 - 2x_3 + 10x_4 = -9$	10	L3	CO1
<b>Module - 2</b>					
Q.3	a.	Find the standard matrix for the linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ defined by $T(x, y, z) = (x - 2y, 2x + y)$ .	10	L2	CO2
	b.	Show that the following function defines an inner product of $\mathbb{R}^2$ , where $u = (u_1, u_2)$ and $v = (v_1, v_2)$ , $\langle u, v \rangle = u_1v_1 + 2u_2v_2$	10	L2	CO2
<b>OR</b>					
Q.4	a.	Apply the Gram-Schmidt orthonormalization process to the basis for $\mathbb{R}^3$ shown below $B = \{(1, 1, 0), (1, 2, 0), (0, 1, 2)\}$	10	L2	CO2
	b.	Find a least-square solution of the system $AX = b$ for $A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$	10	L2	CO2
<b>Module - 3</b>					
Q.5	a.	Diagonalize the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ .	10	L3	CO3

	b.	Find the largest eigen value and the eigen vector of the matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ by Power method.	10	L3	CO3																		
<b>OR</b>																							
Q.6	a.	Solve by Jacobi method $\begin{bmatrix} 2 & \sqrt{2} & 4 \\ \sqrt{2} & 6 & \sqrt{2} \\ 4 & \sqrt{2} & 2 \end{bmatrix}$ .	10	L3	CO3																		
	b.	Find singular value decomposition of a matrix $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 1 \end{bmatrix}$ .	10	L3	CO3																		
<b>Module - 4</b>																							
Q.7	a.	Define: (i) Random sampling (ii) Sampling distribution (iii) Statistical hypothesis (iv) Null hypothesis (v) Level of significance	10	L2	CO4																		
	b.	Five dice were thrown 96 times and the numbers 1, 2 or 3 appearing on the face of the dice follows the frequency distribution as follows: <table border="1" style="margin-left: auto; margin-right: auto;"> <tbody> <tr> <td>No. of dice shown 1, 2 or 3</td> <td>5</td> <td>4</td> <td>3</td> <td>2</td> <td>1</td> <td>0</td> </tr> <tr> <td>Frequency</td> <td>7</td> <td>19</td> <td>35</td> <td>24</td> <td>8</td> <td>3</td> </tr> </tbody> </table> Test the hypothesis that the data follows a binomial distribution ( $\chi_{0.05}^2 = 11.07$ for 5 degree of freedom)	No. of dice shown 1, 2 or 3	5	4	3	2	1	0	Frequency	7	19	35	24	8	3	10	L2	CO4				
No. of dice shown 1, 2 or 3	5	4	3	2	1	0																	
Frequency	7	19	35	24	8	3																	
<b>OR</b>																							
Q.8	a.	A certain stimulus administered to each of 12 patients resulted in the following increases of blood pressure 5, 2, 8, -1, 3, 0, -2, 1, 5, 0, 4, 6. Can it be concluded that the stimulus will increase the blood pressure? (Table value: $t_{0.05}$ for 11 degree of freedom = 2.201)	10	L2	CO4																		
	b.	In order to determine whether there is significant difference in the durability of 3 makes of computers, sample of size 5 are selected from each make and the frequency of repair during the first year of purchase is observed. The results are as follows: Makes <table border="1" style="margin-left: auto; margin-right: auto;"> <tbody> <tr> <td>A</td> <td>5</td> <td>6</td> <td>8</td> <td>9</td> <td>7</td> </tr> <tr> <td>B</td> <td>8</td> <td>10</td> <td>11</td> <td>12</td> <td>4</td> </tr> <tr> <td>C</td> <td>7</td> <td>3</td> <td>5</td> <td>4</td> <td>1</td> </tr> </tbody> </table> In view of the above data test there is a significant difference in the durability of the 3 makes of computers. [ $F_{5\%}(V_1 = 2, V_2 = 12) = 3.88$ ]	A	5	6	8	9	7	B	8	10	11	12	4	C	7	3	5	4	1	10	L2	CO4
A	5	6	8	9	7																		
B	8	10	11	12	4																		
C	7	3	5	4	1																		

Q.9	<p>a. The joint probability distribution of two random variables X and Y is given below:</p> <table border="1" data-bbox="451 232 743 376"> <tr> <td></td> <td>Y</td> <td>-3</td> <td>2</td> <td>4</td> </tr> <tr> <td>X</td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td>1</td> <td></td> <td>0.1</td> <td>0.2</td> <td>0.2</td> </tr> <tr> <td>3</td> <td></td> <td>0.3</td> <td>0.1</td> <td>0.1</td> </tr> </table> <p>Find:                      (i) Marginal distribution of X and Y                      (ii) Covariance of X and Y                      (iii) Correlation of X and Y</p>		Y	-3	2	4	X					1		0.1	0.2	0.2	3		0.3	0.1	0.1	10	L2	CO4
	Y	-3	2	4																				
X																								
1		0.1	0.2	0.2																				
3		0.3	0.1	0.1																				
	<p>b. Show that <math>P = \begin{bmatrix} 0 &amp; 1 &amp; 0 \\ 0 &amp; 0 &amp; 1 \\ \frac{1}{2} &amp; \frac{1}{2} &amp; 0 \end{bmatrix}</math> is a Regular Stochastic Matrix and find the corresponding unique fixed probability vector.</p>	10	L2	CO4																				
<b>OR</b>																								
Q.10	<p>a. A software engineer goes to his workplace every day by motorbike or by car. He never goes by bike on two consecutive days but if he goes by car on a day he is equally likely to go by car or by bike on the next day. Find the transition matrix for the chain of the mode of transport he uses. If car is used on the first day of the week, find the probability that bike is used after 4 days.</p>	10	L2	CO4																				
	<p>b. Cars arrive at a petrol pump, having one petrol unit in Poisson fashion with an average of 10 cars per hour. The service time is distributed exponentially with a mean of 3 minutes. Find:                      (i) Average number of cars in the system                      (ii) Average waiting time in the queue                      (iii) Average queue length                      (iv) The probability that the number of cars in the system is 2.</p>	10	L2	CO4																				

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